Prediction of intrinsic fracture toughness for brittle materials from the apparent toughness of notched-crack specimen

WANG FENGHUI
Department of Engineering Mechanics, Northwestern Polytechnical University, Xi' an 710072, People's Republic of China
E-mail: wfh@pub.xaonline.com

In this investigation, fracture process zone model is used to establish a new relationship to predict the intrinsic fracture toughness from the apparent fracture toughness of a notched-crack specimen. The parameters needed in the proposed model are very rare, such as, the fracture process zone size of materials, the notch radius. Specimens made up of two kinds of polycrystalline alumina and one soda-lime glass with notch radii as small as a few micrometer are used to verify the predictions of this model. Besides, the results also show that fracture toughness of ceramics decreases with the decreasing of notch root radius. Under condition of the radius of crack tip is not greater than the averaged grain size, the apparent toughness can be approximately regarded as the fracture toughness of the materials.

1. Introduction

Fracture toughness is a very important parameter in characterizing mechanical properties of brittle materials, a number of test procedures have been developed to determine the plane strain fracture toughness K_{Ic} of brittle materials. The methods, such as, single edge notched bend (SENB), compact tension (CT), the double cantilever beam (DCB), double torsion (DT), and chevron notch (CHV) specimen are often used in the testing of fracture toughness of brittle materials, in addition to the consideration of the usually machining and measuring conditions [1, 2]. However, the pre-crack used in above test is in the form of a diamond sawed notch due to the using of diamond saw in creating the crack, this, of course, is at varying levels different from the requirement needed by linear elastic fracture mechanics.

Because of the very limited extent of plastic deformation at the notch tip of brittle materials, the traditional pre-cracking method–metallurgical cyclic fatigue technique, is inconvenient to be used to create an ideal crack for ceramics and glass. Therefore, other pre-cracking techniques have been introduced to produce sharp crack ahead of notch tip, such as, bridge indentation [3], compressive cyclic fatigue [4], pressing wedge [5], pre-cracked before sintering [6, 7], stress corrosion [8] and polishing [9], etc. Among them, the indentation method is the most simple one, however, undesirable residual stress around crack is always introduced by this approach. SENB specimen is most rather conventional and widely used one, the advantage of it's simple geometry makes the machining very easy, on the other hand, a small piece of material is enough for specimen. The difficulty for this specimen method is to introduce a natural crack ahead of the sawed notch. Experiments [10–12] revealed that the apparent toughness values that calculated from the maximum load varies with the notch tip radius, the apparent values decrease with decreasing of notch radii first, and then approaches to a constant as the radius of notch tip reaches to a critical value, which seems to be dependent on the microstructure of the materials, especially the grain size [13]. So, it is necessary to asses the effect of the notch radius as well as microstructure of the material on the fracture toughness of brittle materials. Up to now, though some efforts have been made to reveal the relationship between fracture toughness and notch radius, the problem is not solved completely. Usami [14] proposed a grain fracture model, in which he assumed that crack starts ahead of an original crack tip in a large grain whose grain boundary is close to the plane maximum principal stress, so, the apparent fracture toughness tends to be constant when the notch radius is small than three times of average grain diameter. Hishide [15] modified the above model, and assumed that when the elastic strain energy stored in one grain just ahead of the crack tip exceeds the energy of a single crystal, the crack will propagate. Obviously, these model is suitable to describe trans-granular crack.

The main purpose of this paper is to investigate the effect of notch radius on the fracture toughness of brittle materials in much detail experimentally and theoretically, and establish a more philosophically mechanical model to predict fracture toughness of brittle materials from the apparent toughness from sawed notch crack specimen.

2. Assessment model

2.1. Fracture process zone

Brittle materials, such as, ceramics and glasses, is lack of plastic deformation. Experimentally, in order to avoid significant stress singularity at the crack tip, several models, such as, the COD model [16], critical process zone model [17], average stress model [18], were put forward to study the fracture behavior of ceramics. However, the base of those above models was established on the fracture criterion of ductile materials. So it may be worthwhile to develop an actual model brittle material itself for predicting the fracture toughness of accurately.

According to Irwin [19], the principal stress distribution ahead of the crack tip is as follows

$$\begin{aligned}\sigma_{xx} &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\\ \sigma_{yy} &= \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\\ \sigma_{xy} &= \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\end{aligned} \tag{1}$$

where $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ denote principal stresses. Equation 1 shows that the stress in small region ahead of the crack tip is in multi-axial stress state.

For the brittle materials, there also exit a fracture process zone ahead of the crack tip, the stress in the process zone is approximately equal to the fracture stress [16], therefore the process zone can be thought as the rupture-origin zone, but it is difficult to know at which point the fracture will origin. In order to specify the dependent of fracture on the process zone, the average stress over the process zone is considered to be a characteristic parameter. Thus the crack is postulated to grow when the average stress in the process zone exceeds the strength limit. The average stress fracture model can be expressed as

$$\frac{1}{r_0}\int_0^{r_0} \sigma(r)\,\mathrm{d}r = \sigma_s \tag{2}$$

Let $K_I = K_{IC}$ and along the direction $\theta = 0$ in Equation 1, the maximum principal stress is

$$\sigma_{yy} = \frac{K_{IC}}{\sqrt{2\pi r}} \tag{3}$$

Substituting Equation 3 into Equation 2, it yields

$$r_0 = \frac{2}{\pi}\left(\frac{K_{IC}}{\sigma_s}\right)^2 \tag{4}$$

r_0 represents the size of fracture process zone ahead of crack tip at-axis.

2.2. Effect notch radius on fracture toughness

For notched specimen, the stress distributions ahead of notch tip is [20]

$$\begin{aligned}\sigma_{yy} &= \frac{\sigma}{4}\sqrt{\frac{a}{2r+\rho}} \times \left[\left(5\cos\frac{\theta}{2} - \cos\frac{5\theta}{2}\right) + \frac{4\rho}{2r+\rho}\cos\frac{3\theta}{2}\right]\\ \sigma_{xx} &= \frac{\sigma}{4}\sqrt{\frac{a}{2r+\rho}} \times \left[\left(3\cos\frac{\theta}{2} + \cos\frac{5\theta}{2}\right) - \frac{4\rho}{2r+\rho}\cos\frac{3\theta}{2}\right]\\ \sigma_{xy} &= -\frac{\sigma}{4}\sqrt{\frac{a}{2r+\rho}} \times \left[\left(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}\right) + \frac{4\rho}{2r+\rho}\sin\frac{3\theta}{2}\right]\end{aligned} \tag{5}$$

where p and a denote notch radius and notch depth respectively, the origin point of r is at crack tip.

Along the direction $\theta = 0$, the first term of Equation 5 becomes

$$\sigma_{yy} = \frac{K_{In}}{\sqrt{\pi(2r+\rho)}}\left(1 + \frac{\rho}{2r+\rho}\right) \tag{6}$$

where $K_{In} = \sigma\sqrt{\pi a}$, a is notch depth. Substituting (6) into (2), the criterion for notched specimen can be written as

$$\sigma_s = \frac{2K_{In}}{\sqrt{\pi(2r_0+\rho)}} \tag{7}$$

For cracked specimen, the stress around the mode crack can be expressed as [21]

$$\sigma(r) = \frac{\sigma(a+r)}{[r(2a+r)]^{1/2}} \tag{8}$$

Substituting Equation 8 into Equation 2, the criterion becomes

$$\sigma_s = \frac{K_{IC}}{\sqrt{\pi}}\left(\frac{1}{a} + \frac{2}{r_0}\right)^{\frac{1}{2}} \tag{9}$$

where $K_{IC} = \sigma\sqrt{\pi a}$, a denotes depth of crack. In standard tests, the condition $a \gg r_0$ is usually satisfied, then Equation 9 becomes

$$\sigma_s = \sqrt{\frac{2}{\pi r_0}}K_{IC} \tag{10}$$

Combining (7) and (10), the correlation between the fracture toughness K_{Ic} and the apparent fracture toughness K_{In} measured by notched specimen can be obtained as follows

$$\frac{K_{In}}{K_{IC}} = \sqrt{1 + \frac{\rho}{2r_0}} \tag{11}$$

It can be seen from equation 11 that K_{In} approaches to K_{Ic} as the radius of the notch tip reaches zero.

3. Experimental verification

3.1. Experimental procedure

Single edge notched bend specimen with the dimension of $2 \times 5 \times 40$ mm are used for this test. The materials are two kinds of commercial alumina (95%Ai_2O_3) which are marked as Alumina1 and Alumina2, respectively, and one soda-lime glass. The average grain diameter is 20 μm for Alumina1 and 13 μm for Alumina2 (Xi' an Microwave Co), and the strength for Alimina1, Alumina2 and Glass are 249 Mpa, 235 Mpa and 71 Mpa, respectively. The difference of the mechanical property of Alumina1 and Alumina2 is due to the different sintering additives. The straight notch was introduced at the center of the tension surface by means of a diamond cutter, the notch width is 0.26–0.30 mm, then the sharp notch was introduced at the sawed notch tip by using a razor sprinkled with 1.5 um diamond paste. From the mentioned method, the minimum sharp notch with notch radii about 5–6 μm were obtained. The Scan Electron Microscope photographs of the sharp notches are shown in Fig. 1. The relative crack length defined as the ratio of crack length to specimen width, a/w, measured on both sides of specimen is within the range of 0.45–0.55. Bending tests were carried out on a general electric testing machine, the cross-head speed is 0.5 mm/min, the span s between two supporting points was taken to be 20 mm. According to ASTM standard E 399–81 [22], the apparent fracture toughness is calculated by following equation

$$K_{IC} = \left(\frac{P_C}{BW^{1/2}}\right) Y\left(\frac{a}{W}\right)$$

$$Y(\alpha) = \left(\frac{s}{w}\right)\left\{\frac{3\alpha^{1/2}}{2(1+2\alpha)(1-\alpha)^{3/2}}\right\} \times \{[1.99 - \alpha(1-\alpha)(2.15 - 3.93\alpha + 2.7\alpha^2]\} \quad (12)$$

where P_c denotes the peak load at which the notched specimen finally fractures, $\alpha = a/w$. B and W are the thickness and width of the specimen, respectively.

3.2. Experiment results

Fig. 2 shows the testing results of two kinds of alumina and a soda-lime glass, where the tendency of linear regression are represented by the solid line. As can be seen, the fracture toughness decreases with the decreasing of notch tip radius. The apparent fracture toughness measured by using notched specimen may be taken as the fracture toughness K_{Ic} when the notch tip radius is reduced to be equal to the average grain diameter of alumina. Therefore the average grain diameter of alumina may be taken as an estimate of the critical radius of crack tip, below which the values of fracture toughness measured is effectively constant.

The normalized fracture toughness, K_{In}/K_{IC}, is plotted vs the ratio of notch radius to the fracture process

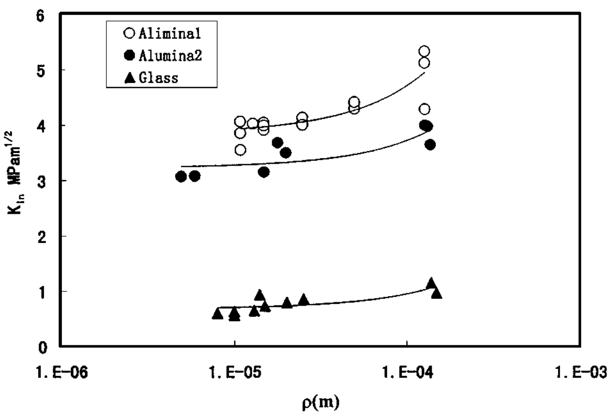

Figure 2 Decreasing apparent fracture toughness as notch radius reduced for alumina1, alumina2 and glass.

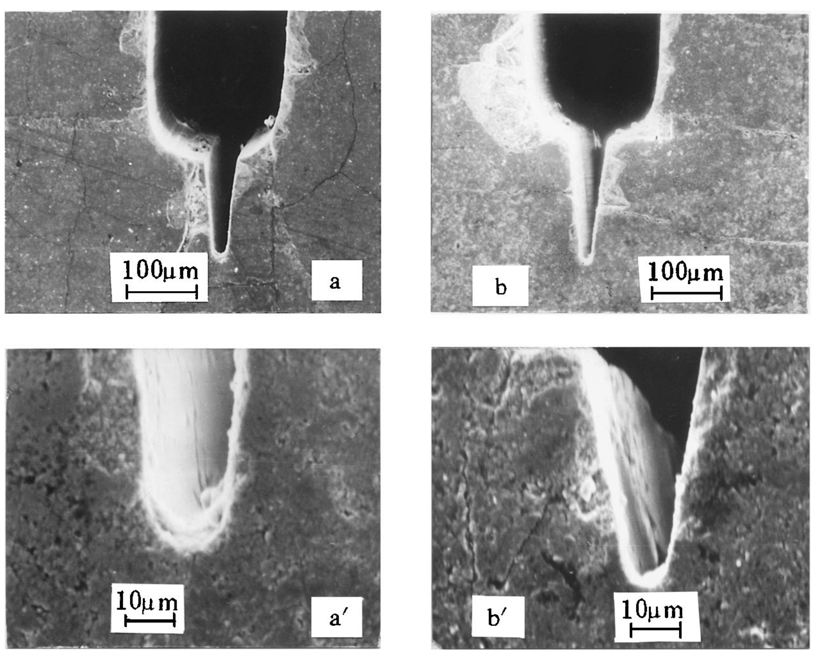

Figure 1 Scan electron microscope photographs of sharp notches, (a) and (a′) $\rho = 8$ μm, (b) and (b′) $\rho = 5$ μm.

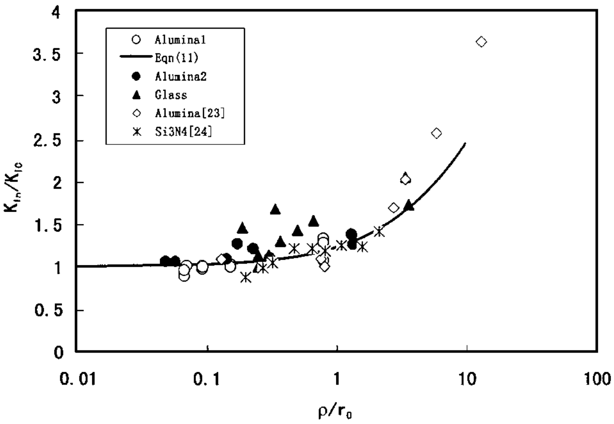

Figure 3 Normalized fracture toughness vs notch radius to process zone size.

zone as shown in Fig. 3, the process zone size are calculated from Equation 4 with the values of fracture toughness and fracture strength, where predicted line (solid line) is drawn according to Equation 11 for two kinds of alumina and the soda-lime glass, the values K_{IC} are extrapolation from zero notch radius. The data from literature [23, 24] are also plotted in Fig. 3. It can been seen that Equation 11 agree well with test results of two kinds of polycrystalline alumina and the data of alumina [23] and silicon nitride [24] from literature, but some points from glass seem large deviation from the solid line, this is probably because that when the glass specimen were machined, the high speed diamond cut cause some piece of glass at the notch tip away from the notch tip, thus resulted in the large deviation of the experiment results.

4. Conclusion

1. The fracture process zone model is developed in this study. The zone size can be used as a characteristic microstructure parameter in predicting the fracture toughness of brittle materials.

2. The apparent fracture toughness decrease with the decreasing notch radius, there exists a critical radius of crack tip, below which the value of fracture toughness is effectively constant and may taken as that of brittle materials. For polycrystalline ceramics, the average grain diameter may be taken as an estimate of the critical radius of crack tip.

3. From the proposed expression, the true fracture toughness K_{IC} can be predicted from test results of apparent fracture toughness measured by using notched specimen from Equation 11.

4. The so-called razor polishing method may be an effective one for pre-cracking in measuring accurate fracture toughness of brittle materials.

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